Recitation 3

September 10

Problem 1. Yes, no, no.

Problem 2. There are seven columns, and they correspond to vectors in $\mathbb{R}^5$. Since $7 > 5$, any seven vectors in $\mathbb{R}^5$ are always linearly dependent. Matrix should have 5 pivot columns for them to span all $\mathbb{R}^5$.

Problem 3. A map $T\colon \mathbb{R}^n \to \mathbb{R}^m$ satisfying $T(cx) = cT(x)$ and $T(x+y) = T(x) + T(y)$ for any scalar $c \in \mathbb{R}$ and any two vectors $x, y \in \mathbb{R}^n$.

Problem 4. T, Z and Q are linear transformations, and F, S are not.

Problem 5.

- $\mathbb{R}^2 \to \mathbb{R}^2$. Both one-to-one and onto.
- $\mathbb{R}^2 \to \mathbb{R}^2$. Neither one-to-one nor onto. Vectors $x_2 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- $\mathbb{R}^3 \to \mathbb{R}^2$. Onto, but not one-to-one. Vectors of the form $x_3 \cdot \begin{bmatrix} -33 \\ -8 \\ 1 \end{bmatrix}$ are killed.
- $\mathbb{R}^2 \to \mathbb{R}^2$. Not onto, but is one-to-one.
- $\mathbb{R}^2 \to \mathbb{R}^2$. Not onto, but is one-to-one.
- $\mathbb{R}^3 \to \mathbb{R}$. Onto, not one-to-one. Vectors $x_2 \cdot \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_3 \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ are killed.

Problem 6. $\begin{bmatrix} 0 & 1 & -3 & 0 \\ -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 \end{bmatrix}$

Problem 7. $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

Problem 8. $\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$

Problem 9. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Problem 10. $\begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$

Problem 11. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$